Intrinsic torsional reorientations in a twisted nematic liquid crystal cell

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The nature of the orientational relaxation process of the director $\hat{\mathbf{n}}$ to its equilibrium orientation $\hat{\mathbf{n}}_{eq}$, in the twisted nematic cell, under the influence of an external electric field, is investigated. The influence of the electric, elastic, and viscous torques on the dynamics of the director is reflected in the relaxation of the director $\hat{\mathbf{n}}$ to $\hat{\mathbf{n}}_{eq}$, with different relaxation times. It is shown that the relaxation time, both for the cases of a strong and weak anchoring, exhibits an anomalous increase with decreasing of an external electric field, whereas the influence of the azimuthal anchoring energy, in the case of the twisted nematic cell is characterized by a weak effect. It is also shown that these torques exerted on the director may excite the traveling wave spreading from one edge of the cell to their second edge. Calculations of the relaxation processes in the vicinity of a nematic—smectic-*A* (NA) phase transition temperature T_{NA}, e.g., at a few tens of mK from T_{NA} in the nematic phase, shows that the director distortion in the gap between two plates is maintained to be constant across the sample both in the case of a strong and weak anchoring.

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I. INTRODUCTION

Uniform textures of twisted nematics (TN) are produced by orienting a drop of bulk material in between two conveniently treated plates, which define usually a fixed orientation for the boundary molecules. This is the so-called strong anchoring orientation. Applying an electric field $\mathbf{E}$ parallel to an uniformly oriented TN can distort the molecular orientation $\hat{\mathbf{a}}$ with respect to director $\hat{\mathbf{n}}$, at a critical field E_{th} given by [1]

$$E_{th} = \frac{\pi}{d}\sqrt{\frac{K_2}{\epsilon_0 \epsilon_a}}, \tag{1}$$

where d is the sample thickness, K_2 is the twist elastic constant, ϵ_0 is the absolute dielectric permittivity of free space, $\epsilon_a = \epsilon_{\parallel} - \epsilon_{\perp}$ is the dielectric anisotropy of the TN, $\epsilon_{\parallel}$ and $\epsilon_{\perp}$ are the dielectric constants parallel and perpendicular to the director $\hat{\mathbf{n}}$, respectively. This form for the critical field is based upon assumptions that the director remains strongly anchored (in our case, homogeneously) at the two surfaces and that alignment of the director $\hat{\mathbf{n}}$ is uniformly constant across the sample for $E < E_{th}$. On the other hand, with increase an external field $E > E_{th}$, the theoretical description of dissipation processes in a TN confined at its end by fixed plates which align the rodlike molecules along a preferred in-plane direction (directed, for instance, parallel to the x axis), exhibit a number of regimes of relaxation. These regimes, in which the director rotates in the plane parallel to both glass plates, exert a torque directed to be normal to these boundaries. They are, however, thermodynamically metastable, because an external electric field is directed to be parallel to restricted plates (parallel to the y axis), and large orientation fluctuation may occur which carry the system between metastable and stable states, relieving and enhancing the twist. Thus, if a constant torque is maintained externally at the end plates, twist reducing fluctuations may relax the external torque which was balancing the torque applied to the plates. In response, the system will twist, thus increasing the internal torque until it again balances the applied torque. Moreover, for narrow (or ultranarrow) TN cells ($d < 2$-3 μm) anchoring plays a major role and the effects of surfaces on the relaxation process need to be examined especially at temperatures close to a nematic—smectic-*A* (NA) second-order transition T_{NA}, because it leads to introduction of smectic positional molecular order in the boundary region [2,3]. Existence of Sm*A* structures induced by the surface in nematic phase of twisted liquid crystals (LCs) infer a surface induced smectic density wave decaying exponentially into the liquidlike nematic bulk [4–6]. In turn, the growth of pretransitional Sm*A* fluctuations are expected to give rise to a novel torque on $\hat{\mathbf{n}}$, which alters the viscous torque. As a result, the effect of fluctuations is reflected in a renormalization of the viscous and some elastic coefficients.

So, the problem of predicting of the relaxation processes in a twisted nematic cell is far from being trivial, and more realistic theoretical treatments which can elucidate the role of an external electric field, anchoring conditions and temperature are needed.

The outline of this article is as follows: a dynamic equation describing the reorientation of a director in the twisted nematic cell and numerical calculation of the relaxation of the director to its equilibrium orientation, for number of regimes of relaxation, both in the vicinity of the second-order nematic—smectic-A phase transition temperature and far from it, are given in Sec. II. Conclusions are summarized in Sec. III.

II. SURFACE REORIENTATION DYNAMICS OF NEMATIC LIQUID CRYSTALS

Here we localize our attention on the azimuthal anchoring, when the polar angle θ is fixed, and the anchoring en-

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ergy is a function of the surface azimuthal angle ϕ only. The dynamic equation describing the reorientation of the director distortion in the gap between two glass plates is maintained by elastic, electric and viscous torques as

$$\mathbf{T}_{el} + \mathbf{T}_{elast} + \mathbf{T}_{vis} = 0. \quad (2)$$

In the case of planar geometry $\hat{\mathbf{n}} = (\cos\phi(z), \sin\phi(z), 0)$ and absence of flow, the viscous, electric and elastic torques takes the form $\mathbf{T}_{vis} = -\gamma_1 \hat{\mathbf{n}} \times (\partial \hat{\mathbf{n}} / \partial t)$, $\mathbf{T}_{el} = \epsilon_0 \epsilon_a \hat{\mathbf{n}} \times \mathbf{E}(\hat{\mathbf{n}} \cdot \mathbf{E})$, and $\mathbf{T}_{elast} = \hat{\mathbf{n}} \times \mathbf{h}_t$, respectively. Here $\mathbf{h}_t = -K_2[\mathcal{A}\nabla \times \hat{\mathbf{n}} + \nabla \times (\mathcal{A}\hat{\mathbf{n}})]$, and $\mathcal{A} = \hat{\mathbf{n}} \cdot (\nabla \hat{\mathbf{n}})$ is the twist component of the molecular field, γ_1 is the rotational viscosity coefficient (RVC), $\epsilon_a = \epsilon_\parallel - \epsilon_\perp$, $\epsilon_\parallel$ and $\epsilon_\perp$ are the dielectric constants parallel and perpendicular to the director $\hat{\mathbf{n}}$, and ϵ_0 is the dielectric permittivity of free space. In the case of the planar geometry, when $\hat{\mathbf{n}}$ always remaining in the plane of the plates [x-y plane defined by the director $\hat{\mathbf{n}}_s$ on the lower plate (x direction) and the electric field direction (y direction); $\hat{\mathbf{k}} = \hat{\mathbf{i}} \times \hat{\mathbf{j}}$ is directed to be normal to both glass plates], $\phi(z)$ denotes the azimuthal angle, i.e., the angle between the direction of the unit vector $\hat{\mathbf{i}}$ and the director $\hat{\mathbf{n}}$. The torque due to electric field is given by $\mathbf{T}_{el} = (E^2/2)\epsilon_0 \epsilon_a \sin 2\phi(t,z)\hat{\mathbf{k}}$, whereas the viscous torque takes the form $\mathbf{T}_{vis} = \gamma_1 \partial_t \phi(t,z)\hat{\mathbf{k}}$, where $\partial_t \phi(t,z) = \partial\phi(t,z)/\partial t$. The torque due to elastic forces is $\mathbf{T}_{elast} = K_2 \partial_{zz}\phi(t,z)\hat{\mathbf{k}}$, where $\partial_{zz}\phi(t,z) = \partial^2\phi(t,z)/\partial z^2$.

Taking this into account Eq. (2) can be written in a dimensionless form as

$$\partial_\tau \phi(\tau,z) = \partial_{zz}\phi(\tau,z) + \delta \sin 2\phi(\tau,z), \quad (3)$$

where $\tau = K_2 t/\gamma_1 d^2$, $\delta = (\pi^2/2)(E/E_{th})^2$, and $z = z/d$ is the dimensionless direction through the cell thickness. Here $E_{th} = (\pi/d)\sqrt{K_2/\epsilon_0\epsilon_a}$ is the Freedericksz threshold field in this geometry. The partial differential Eq. (3) describing the dynamics of director in the twisted cell will later be solved numerically, however the static equilibrium equation may be found analytically by solving the time independent equation for $\phi(z)$

$$\partial_{zz}\phi_{eq}(z) + \delta \sin 2\phi_{eq}(z) = 0. \quad (4)$$

A. Case of strong anchoring

In the case of the strong anchoring the boundary conditions are

$$\phi(z)_{z=0} = 0, \quad \phi(z)_{z=1} = 0, \quad (5)$$

whereas the initial orientation of the director is disturbed parallel to the external field $\mathbf{E}$, with $\phi(\tau = 0, z) = \pi/2$, and then allowed to relax to its equilibrium value $\phi_{eq}(z)$. The solution (4) with the boundary conditions (5) is [7]

$$z = 2\delta \int_0^\psi \frac{d\lambda}{\sqrt{1 - \sin^2\phi_m \sin^2\lambda}} = 2\delta \mathcal{K}(\psi, \sin\phi_m), \quad 0 \leq z \leq \frac{1}{2}, \quad (6)$$

where $\mathcal{K}(\psi, k)$, $k = \sin\phi_m$ is the elliptic integral of the first kind having modulus k, $\phi_m = \phi(\frac{1}{2})$. The solution for $\frac{1}{2} < z \leq 1$ is obtained from Eq. (6) by means of the simple replacing z by $1-z$. The relaxation of the director $\hat{\mathbf{n}}$ to its equilibrium orientation, which is described by the angle $\phi(\tau,z)$ from the initial condition $\phi(\tau = 0, z) = \pi/2$ to $\phi_{eq}(z)$, and the planar director alignment on both surfaces [conditions (5)], at different values of $\delta = (\pi^2/2)(E/E_{th})^2$, have been investigated by a standard numerical relaxation method [8] and results are shown in Figs. 1 and 2. It is found that the values of $\phi(\tau,z)$, in the case of $E/E_{th} \leq 1.0$ [case (1)], relaxed to zero, whereas, in the case of $E/E_{th} = 1.01$ [case (2)], the azimuthal angle $\phi(\tau,z)$ relaxed to small equilibrium angle $\phi_{eq}(z)$, and the values of $\phi_{eq}(z)$ vary slow between 0, at the boundary of the cell, and 0.2 ($\sim 11.5°$), in the center of the cell. The relaxation criterion $\kappa = |(\phi(\tau_R) - \phi_{eq})/\phi_{eq}|$ for calculating procedure was chosen equal to 10^{-4}, and the numerical procedure was then carried out until a prescribed accuracy was achieved.

It is important to stress that the balance between the electric, elastic and hydrodynamic torques exerted on the director is reflected in the growth of the relaxation time

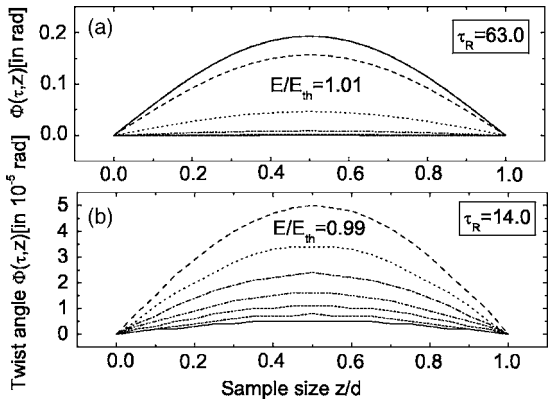

FIG. 1. (a) Plot of relaxation of the azimuthal angle $\phi(\tau,z)$ ($\tau = K_2 t/\gamma_1 d^2$ is a dimensionless time) to its equilibrium value $\phi_{eq}(z)$ (solid line) in the twisted nematic cell, calculated using Eq. (3), with the planar boundary conditions (5), at $E/E_{th} = 1.01$. (b) Same as (a), but the angle $\phi(\tau,z)$ is calculated using Eq. (3), with the value of $E/E_{th} = 0.99$.

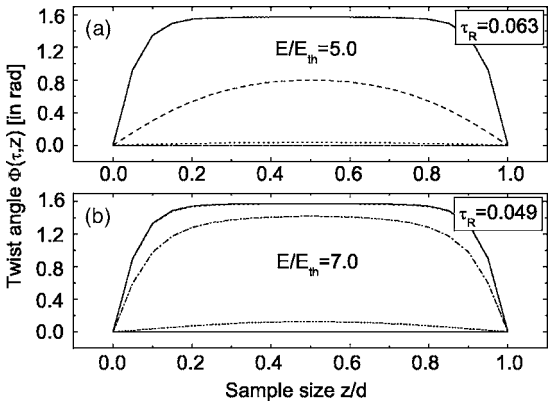

FIG. 2. Same as Fig. 1, but the angle $\phi(\tau,z)$ is calculated using Eq. (3), with the boundary conditions (5), at $E/E_{th} = 5.0$ (a) and 7.0 (b), respectively.

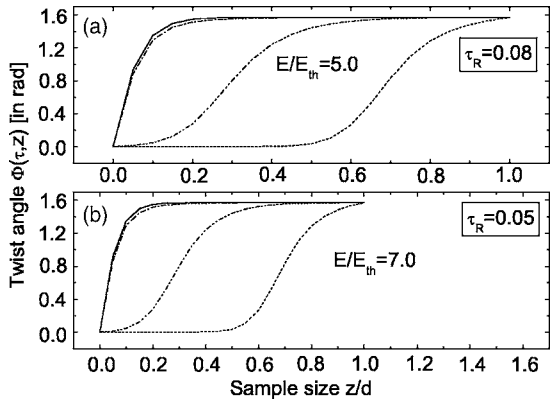

FIG. 3. Same as Fig. 1, but the angle $\phi(\tau,z)$ is calculated using Eq. (3), with the boundary conditions (7), at $E/E_{th}=5.0$ (a) and $E/E_{th}=7.0$ (b), respectively.

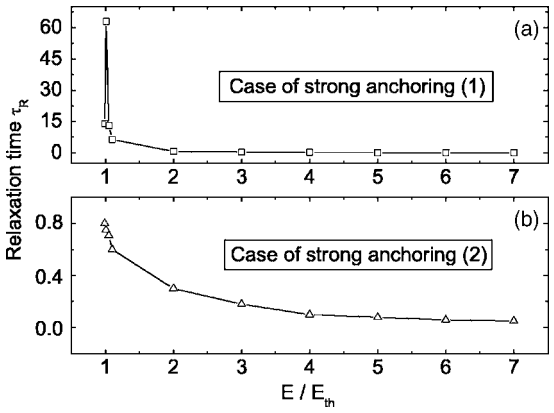

FIG. 4. Influence of the external field E on the relaxation time τ_R. The relaxation time dependencies of the function E/E_{th}, with the boundary conditions (5) [case (1)] (a) and (7) [case (2)] (b), respectively.

$\tau_R=(\gamma_1 d^2/K_2)t_R$, in case (2), approximately four times larger than in case (1). With the increase of E/E_{th} up to 7, the value of the relaxation time τ_R is two order times smaller than at $E/E_{th}=1.01$, and the azimuthal angle $\phi(\tau,z)$ relaxed to equilibrium angle $\phi_{eq}(z)$, which, in turn, vary rapidly (within the first 0.2 layer from the boundary) between 0 and $\pi/2$.

In the case of the boundary conditions

$$\phi(z)_{z=0}=0, \quad \phi(z)_{z=1}=\frac{\pi}{2}, \tag{7}$$

when the director on the upper plate is at right angles to the director on the lower plate, both alignments being within the plane of the plates, and the relaxation of the director to its equilibrium orientation are shown in Fig. 3.

The relaxation processes in the vicinity of the electric field $E/E_{th}=1.01$, for geometry with the boundary conditions (7), is much faster, approximately two order times, than for the same relaxation processes with the boundary conditions (5).

It should be pointed out here that the dimensionless electric field E/E_{th}, for geometry with the boundary condition (7), was also scaled in E_{th} units.

Calculations also show that the effect of the external electric field E/E_{th} on the relaxation time τ_R both for the boundary conditions (5) and (7), decreases as the magnitude of E/E_{th} increases and saturates at $E/E_{th}\sim 6.0$ (see Table I).

The influence of the external electric field E on the relaxation time τ_R of the director $\hat{\mathbf{n}}$ to its equilibrium orientation in the twisted nematic cell, both for the planar alignment with conditions (5) [case (1)] and (7) [case (2)], are shown in Fig. 4.

TABLE I. The orientational relaxation times τ_R calculated using Eq. (3), for the case of strong anchoring, both for the boundary conditions (5) [case (1)] and (7) [case (2)], respectively.

E/E_{th}	0.99	1.01	1.05	1.1	2.0	3.0	4.0	5.0	6.0	7.0
$\tau_R(1)$	14	63	13	6.3	0.6	0.35	0.15	0.063	0.06	0.05
$\tau_R(2)$	0.8	0.75	0.71	0.6	0.3	0.18	0.1	0.08	0.06	0.05

Both these cases (1) and (2) are characterized by increasing of the relaxation time τ_R with decrease the electric field E. Note that the second (2) relaxation regime is characterized by much slower, approximately, two order times the relaxation time, than in the case of the first (1) relaxation regime.

B. Case of weak anchoring

Consider the same situation for the twisted nematic, but now the director $\hat{\mathbf{n}}$ is weakly anchored to both boundary plates and the anchoring energy takes the form [9]

$$W_{az}=W_{az}(\phi_s-\phi_0)=\frac{1}{2}A\sin^2(\phi_s-\phi_0), \tag{8}$$

where A is the anchoring strength, and ϕ_s and ϕ_0 are the azimuthal angles corresponding to the director orientation on the boundary plate and easy axis $\hat{\mathbf{e}}$, respectively. The torques transmitted to the surface are the elastic torque $T_{elast}=(K_2/d)(\partial\phi(z)/\partial z)_{z=0,1}$ tends to align $\hat{\mathbf{n}}_s$ along $\mathbf{E}$, and the opposed anchoring torque $T_{anchor}=-(\partial W/\partial\phi_s)$ rotates $\hat{\mathbf{n}}_s$ toward $\hat{\mathbf{e}}$, and, at least, the surface viscous torque $T_{vis}=-\gamma_s(\partial\phi_s/\partial t)$. At the time scale $t\ll\tau_s=\gamma_s d/(K_2-dA\Delta\phi)$, where $\Delta\phi=\phi_s-\phi_0$, one can neglect the effect of the surface viscosity to the torque balance and the director angle has to satisfy the boundary conditions

$$\frac{K_2}{d}(\partial\phi(z)/\partial z)_{z=0,1}=A\Delta\phi. \tag{9}$$

For the case of 4-*n*-octyl-4′-cyanobiphenyl, (8CB), at $T=308$ K, $K_2=5.84$ pN [10], and for narrow (or ultranarrow) TLC cells $d\sim 2.0$–2.5 μm. For the homogeneously (planar) aligned LCs at an indium tin oxide (ITO) surface, the experimental data for A, obtained using different experimental techniques, are varied between 10^{-4} and 10^{-6} J/m^2, so the combination of Ad/K_2 values varied between 0.43 and 43, respectively. In the case of the small $\Delta\phi$, for instance, $\Delta\phi\in[0.03,0.3]$, the values of $(Ad/K_2)\Delta\phi$ are varied between 0.01 and 4.0. We note that the above $\Delta\phi$ is lower than 10° [11]. The stationary solution of Eq. (4) with the boundary

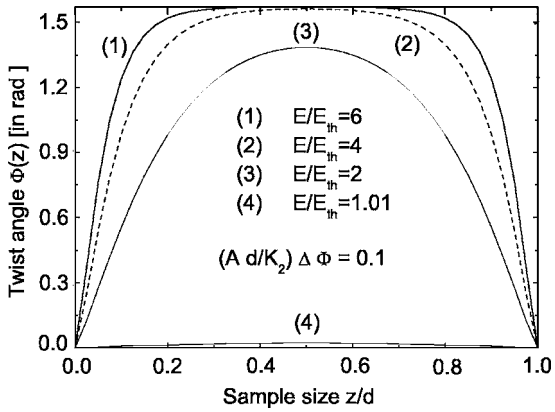

FIG. 5. Plot of the stationary angle $\phi(z)$ ($z=z/d$ is a dimensionless size) in the twisted nematic cell, calculated using Eq. (4), with the boundary conditions (9), at the $(Ad/K_2)\Delta\phi=0.1$, and for a number of values of $E/E_{th}=6.0$ [curve (1)], 4.0 [curve (2)], 2.0 [curve (3)], and 1.01 [curve (4)], respectively.

conditions (9) have also been investigated by a standard numerical relaxation method and results, for a number of E/E_{th} values: 6.0 [curve (1)], 4.0 [curve (2)], 2.0 [curve (3)], and $\sim$1.0 [curve (4)], and at the $(Ad/K_2)\Delta\phi=0.1$, are shown in Fig. 5.

It is found that the values of $\phi(\tau,z)$, in the case of $E/E_{th}\sim 1.0$ [case (4)] relaxed to zero, whereas, in the case of $E/E_{th}>1.0$ [cases (3), (2), and (1)], the azimuthal angle relaxed to equilibrium angle $\phi_{eq}(z)$, and the values of $\phi_{eq}(z)$ vary rapidly, with increase of E/E_{th} up to 6, within the first 0.13 layer from the boundary, between 0 and $\pi/2$. The relaxation of the director $\hat{\mathbf{n}}$ to its equilibrium orientation, which described by the angle $\phi(\tau,z)$ from the initial condition (see the lowest curves in Fig. 6) to $\phi_{eq}(z)$, with the boundary conditions (9), at different values of $(Ad/K_2)\Delta\phi=0.01$ and 0.1, and the value of $E/E_{th}=3.0$, have been investigated by a standard numerical relaxation method and results are shown in Fig. 6.

With the increase of $(Ad/K_2)\Delta\phi$ from 0.01 up to 0.1, the value of the relaxation time τ_R vary slow between 0.075 and 0.125, respectively. The relaxation of the director $\hat{\mathbf{n}}$ to its

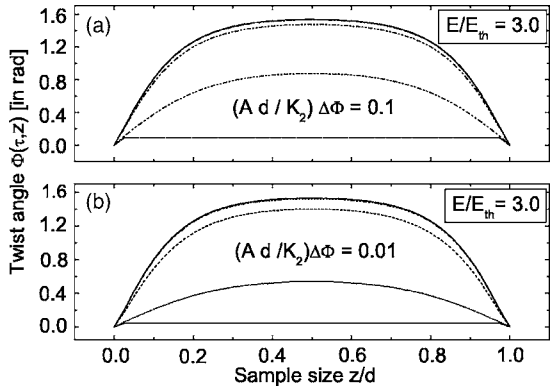

FIG. 6. (a) Same as Fig. 1, but the angle $\phi(\tau,z)$ is calculated using Eq. (3), with the boundary conditions (9), at $E/E_{th}=3.0$ and $(Ad/K_2)\Delta\phi=0.1$. (b) Same as (a), with the value of $E/E_{th}=3.0$ and $(Ad/K_2)\Delta\phi=0.01$.

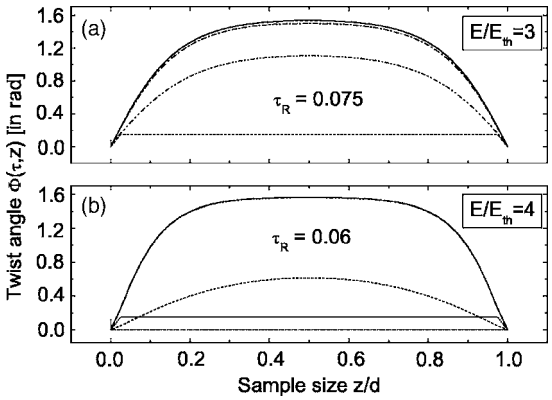

FIG. 7. (a) Same as Fig. 1, but the angle $\phi(z,\tau)$ is calculated using Eq. (3), with the boundary conditions (9), at $E/E_{th}=3.0$ and $A\Delta\phi=0.1$. (b) Same as (a), but for $E/E_{th}=4.0$ and $(Ad/K_2)\Delta\phi=0.01$.

equilibrium orientation, at different values of $E/E_{th}=3.0$ and 4.0, are shown in Fig. 7.

With the increase of E/E_{th}, at the same value of $(Ad/K_2)\Delta\phi=0.1$, the value of the relaxation time τ_R vary slow between 0.075 and 0.06. The influence of the external electric field E on the relaxation process of the director to its equilibrium orientation in the twisted nematic cell, with boundary conditions (9) is shown in Fig. 8(b).

With the increase of E/E_{th} from $\sim$1.0 up to 4.0, the value of the relaxation time τ_R is order times smaller than at $E/E_{th}\sim 1.0$. The influence of the anchoring strength A on the relaxation time τ_R of the director to its equilibrium orientation in the twisted nematic cell, with the boundary conditions (9), is shown in Fig. 8(a). The electric field $(E/E_{th}=4.0)$ puts the director into an equilibrium orientation with, practically, the same relaxation times τ_R, which decrease slow with increasing anchoring strength: 0.075 for $(Ad/K_2)\Delta\phi=0.01$, and 0.05 for $(Ad/K_2)\Delta\phi=4.0$, respectively. Having obtained the function $\phi(\tau,z)$, one can determine the angular velocity $\omega(\tau,z)=\partial\phi(\tau,z)/\partial\tau$ of the director $\hat{\mathbf{n}}$ in the twisted nematic

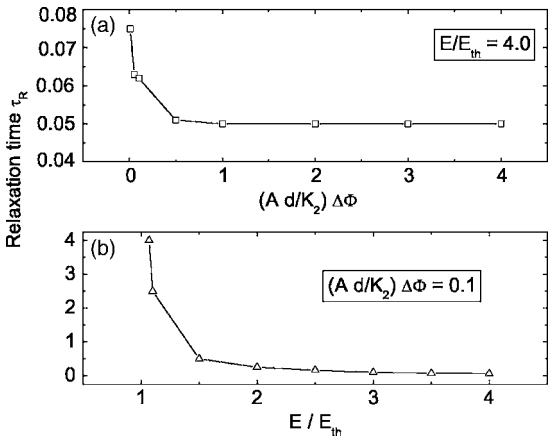

FIG. 8. (a) Influence of the strength of anchoring energy $(Ad/K_2)\Delta\phi$ on the relaxation time τ_R, calculated using Eq. (3), with the boundary conditions (9), at $E/E_{th}=4.0$. (b) Influence of the external field E on the relaxation time τ_R, calculated using Eq. (3), with the boundary conditions (9), at $(Ad/K_2)\Delta\phi=0.1$.

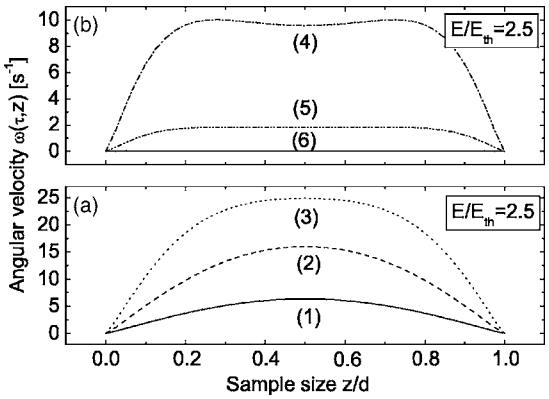

FIG. 9. (a) Plot of the angular velocity $\omega(\tau,z)$ of the director $\hat{\mathbf{n}}(\tau,z)$ in the TN cell, calculated using Eq. (3), with boundary conditions (9), at $E/E_{th}=2.5$ and $(Ad/K_2)\Delta\phi=0.1$, for number of times (1) $\tau_1=0.04$, (2) $\tau_2=0.05$, and (3) $\tau_3=0.06$. (b) Same as (a), but the sequences of the times are (4) $\tau_4=0.075$, (5) $\tau_5=0.09$, and (6) $\tau_6=\tau_R=0.15$, respectively.

cell. Calculations of the magnitude of $\omega(\tau,z)$ shows that under the external electric field $E/E_{th}=2.5$ (Fig. 9), the angular velocity of the director is characterized by increasing of $\omega(\tau,z)$ up to 25[s^{-1}] within the first half of the relaxation term ($\sim$0.06), and fast decreasing of $\omega(\tau,z)$ up to zero, within the second half of the relaxation term ($\sim$0.15), respectively. Note that the second half of the relaxation regime is characterized by a complicate behavior of the $\omega(\tau,z)$. The values of $\omega(\tau,z)$ corresponding to the $\tau=\tau_4$ [see Fig. 9(b)] shows that the highest magnitude of the angular velocity $\omega(\tau_4,z)$ are realized, approximately, at the half distance between both boundaries and the center of the cell.

C. Case of traveling wave solution

We are interested now in the front propagation in systems with the viscous dissipation, and governing by dynamic Kolmogorov-Fisher equation [12,13]

$$\gamma_1\frac{\partial\phi(t,z)}{\partial t}=K_2\frac{\partial^2\phi(t,z)}{\partial z^2}+\Delta\sin 2\phi(t,z), \tag{10}$$

where $\Delta=\epsilon_0\epsilon_a E^2/2$.

Because, in the our case, the field $\mathbf{E}$ is aligned parallel to the y direction, the state $\phi_{z=0}(z)=0$ is now unstable, and front $\phi(t,z)$ starts to move away from the one edge ($z=0$) of the cell to their second edge ($z=1$). Its velocity v to be determined by the balance of the elastic, electric, and hydrodynamic torques. The asymptotic velocity v is essentially determined by a simple dynamical mechanism and $\phi(t=0,z)$ falls off exponentially with a decay length inversely proportional to the field strength E. The front speed v is obtained by substituting

$$\phi(t,z)\sim\exp\left[-E\sqrt{\frac{\epsilon_0\epsilon_a}{K_2}}(z-vt)\right], \tag{11}$$

into linearized form of the Eq. (10), and one sees that the slowest velocity has a value

$$v=2\sqrt{\frac{\epsilon_0\epsilon_a K_2}{\gamma_1^2}}E, \tag{12}$$

and a wave narrowest thickness κ is inversely proportional to the electric field strength E

$$\kappa=\sqrt{\frac{K_2}{\epsilon_0\epsilon_a}}\frac{1}{E}. \tag{13}$$

Hence, if we have $E\geqslant\pi E_{th}$, only then is the wave short enough to fit in the cell length.

D. Temperature close to the T_{NA}

As temperature is reduced towards T_{NA}, growth of pretransitional Sm*A* fluctuations are expected to give rise to a novel torque $\mathbf{T}_{fl}$ on $\hat{\mathbf{n}}$, which alters the $\mathbf{T}_{vis}$. The physical origin of $\mathbf{T}_{fl}$ is due to the effect of shear flow on the fluctuation domains. As a result, the effect of fluctuations, at lowest order, is reflected in renormalization of γ_1 and K_2 [14–16]

$$\bar{\gamma}_1=\gamma_1+\gamma_1^c, \tag{14}$$

and

$$\bar{K}_2=K_2+K_2^c, \tag{15}$$

where $\gamma_1^c=(k_BT/4)(\pi/\xi_0)\sqrt{(\rho_m/K_1)}t^{\nu-1}$. Here K_1 is the splay elastic deformation, ρ_m is the mass density, ξ_0 is the bare correlation length, $t=(T/T_{\mathrm{NA}}-1)$, and $\nu=\nu_\parallel$ is the associated critical exponent. In our case the twist deformation K_2^c can be written in the form [14]

$$K_2^c=\frac{k_BT}{6}\frac{\pi}{l^2}\frac{\xi_\perp^2}{\xi_\parallel}=\frac{k_BT}{6}\frac{\pi}{l^2}\frac{\xi_{0,\perp}^2}{\xi_{0,\parallel}}t^{-2\nu_\perp+\nu_\parallel}. \tag{16}$$

Here l is the layer spacing of the smectic layers, $\xi_\parallel=\xi_{0,\parallel}t^{-\nu_\parallel}$ and $\xi_\perp=\xi_{0,\perp}t^{-\nu_\perp}$ are the longitudinal and transverse correlation lengths, $\xi_{0,\parallel}$ and $\xi_{0,\perp}$ are their background parts, respectively. In the vicinity of T_{NA}

$$\lim_{t\to 0}\frac{\bar{K}_2}{\bar{\gamma}_1}\sim\frac{t^{-2\nu_\perp+\nu_\parallel}}{t^{-1+\nu_\parallel}}\sim t^{1-2\nu_\perp}.$$

In the case of 8CP the value of $\nu_\parallel\sim 0.67$, $\nu_\perp\sim 0.55$, $\xi_{0,\parallel}\sim 0.45$ nm, and $\xi_{0,\perp}\sim 0.2$ nm [4–6], respectively, and one have that the ratio $\lim_{t\to 0}(\bar{K}_2/\bar{\gamma}_1)\to\infty$. As a result, the effect of fluctuations is reflected in the renormalization of γ_1 and K_2 in Eq. (10) to $\bar{\gamma}_1$ and $\bar{K}_2$, respectively. So, in the vicinity of T_{NA}, Eq. (10) takes the form

$$\frac{\partial^2\phi(t,z)}{\partial z^2}=0. \tag{17}$$

Together with the boundary conditions, $\phi_{z=0,1}(z)=0$, one has $\phi=\phi_s(T_{\mathrm{NA}})\equiv 0$. Physically, this means that the alignment of the director $\hat{\mathbf{n}}$ is uniformly constant across the sample as in the case of the strong anchoring without an external field E. For temperatures close to the second-order phase transition T_{NA}, the boundary conditions (9) takes the form

$$\left(\frac{\partial\phi(t,z)}{\partial z}\right)_{z=0,1}=0, \tag{18}$$

and Eq. (17) has a solution

$$\phi(z)=\phi_s(T_{\mathrm{NA}}).$$

Physically, this means that the prescribed alignment of the director $\hat{\mathbf{n}}$ is the same at both plates as well as in the center of the cell. Our analysis also shows that in the vicinity of the $N-\mathrm{Sm}A$ phase transition temperature the role of the viscous and elastic coefficients increases, due to the divergences to infinity both the twist deformation and rotational viscosity coefficients. This result is important because it shows that the relaxation time t_R increase to infinity when the temperature $T\to T_{\mathrm{NA}}$, e.g., at a few tens of mK from T_{NA} in the nematic phase, proportional to $\lim_{t\to 0} t_R=\lim_{t\to 0}(\bar{K}_2/\bar{\gamma}_1 d^2)\tau_R\sim t^{1-2\nu_\perp}$. In the case of 8CB the value of $\nu_\perp\sim 0.55$, and one have that the ratio $\lim_{t\to 0} t_R=\lim_{t\to 0}(\bar{K}_2/\bar{\gamma}_1)\to\infty$.

It should be mentioned that in the vicinity of the second-order phase transition temperature T_{NA}, the predicted velocity $v=0$. Indeed,

$$\lim_{t\to 0} v=\lim_{t\to 0}\left[2\sqrt{\frac{\epsilon_0\epsilon_a K_2}{\gamma_1^2}}E\right]\to 0. \tag{19}$$

So, in the vicinity of the second phase transition temperature, one deal with the stationary case, and the alignment of the director $\hat{\mathbf{n}}$ is uniformly constant $\phi\equiv 0$ across the sample as in the case of the strong anchoring in the presence only the twist elastic deformation.

Probably, it can be fixed experimentally; first of all, let us consider the case when T far from T_{NA}, and the external electric field is applied parallel to the axis y. In the case of the strong anchoring, and the external field $E>E_{th}$, the alignment of the director inside the cell can be described by Eq. (3) with the boundary conditions in the form (4). When the temperature $T\to T_{\mathrm{NA}}$, e.g., at a few tens of mK from T_{NA} in the nematic phase, the director distortion in the gap between two glass plates will maintain to be constant across the sample, and the role of the electric field $\mathbf{E}$ becomes important only at values of E increasing proportional to $t^{-\nu}$, with decreasing t [17].

The torque transmitted to the surface $T_{sur}=(K_2/\xi)\sin\phi_s$ tends to align $\hat{\mathbf{n}}_s$ along $\mathbf{E}$, and an opposed anchoring torque $T_{anch}=-\partial W/\partial\phi_s$ rotates $\hat{\mathbf{n}}_s$ toward $\hat{\mathbf{n}}_0$. Here $\xi=(1/E)\sqrt{K_2/\epsilon_0\epsilon_a}$ is the electric field correlation length. The balance of the torques applied on $\hat{\mathbf{n}}_s$ is

$$T_{sur}+T_{anch}=\frac{K_2}{\xi}\sin\phi_s-\frac{\partial W}{\partial\phi_s}=\frac{K_2}{\xi}\sin\phi_s-\frac{A}{2}\sin 2(\phi_s-\phi_0)=0, \tag{20}$$

where ϕ_s is the director orientation on the surface, whereas ϕ_0 is the easy axis orientation. So, one has

$$A\sin 2\Delta\phi=2\sqrt{K_2\epsilon_0\epsilon_a}E\sin\phi_s, \tag{21}$$

where $\Delta\phi=\phi_s-\phi_0$.

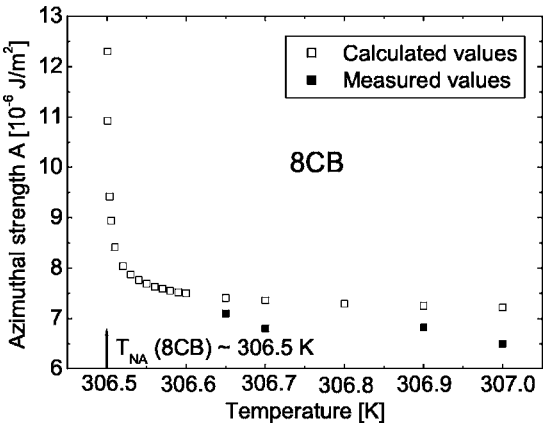

FIG. 10. The temperature dependence of the azimuthal strength of the anchoring energy A, calculated using Eq. (21), and measured values in Ref. [18] of A, respectively.

When the temperature $T\to T_{\mathrm{NA}}$, the azimuthal anchoring energy also increase to infinity, proportional to $t^{-\nu_\perp+\nu_\parallel/2}$, with decreasing t. Recently, the strong increase of azimuthal anchoring coefficient A, using dynamic light scattering, has been observed for 8CB at $\sim$307.0 K [18]. For the 1 μm cell at $T\sim 307$ K, the measured value of the azimuthal strength anchoring energy for 8CB have been found to be $\sim 7\times 10^{-6}$ J/m^2 [18,19]. In order to apply Eq. (21) to our calculation, we use the $\bar{K}_2(8\mathrm{CB})\sim 8$ pN (in the vicinity of NA phase transition temperature $\sim$307 K [10,20]). The thresholds voltage $U_{th}=\pi\sqrt{K_2/\epsilon_0\epsilon_a}$ is equal to be $\sim$0.9 V. Experimentally $\Delta\phi$ is rather small, $\Delta\phi\sim 10°$ [11,21], and therefore $\sin 2\Delta\phi\sim 2\Delta\phi$. Now there is the trigonometrical equation for ϕ_s and one obtains

$$\sin\phi_s=\frac{\Delta\phi A}{\sqrt{K_2\epsilon_0\epsilon_a}E}=\frac{\Delta\phi A}{\sqrt{K_2\epsilon_0\epsilon_a}}\frac{d}{U}.$$

Substituting these data into the last equation one obtains $\sin\phi_s(T_{\mathrm{NA}})\sim 0.05$. So, when the temperature $T\to T_{\mathrm{NA}}$, e.g., at a few tens of mK from T_{NA} in the nematic phase, the director distortion in the gap between two glass plates, with the boundary conditions (9), will maintain to be constant across the sample, with the angle $\lim_{T\to T_{\mathrm{NA}}}\phi_s(T_{\mathrm{NA}})\sim 0°$. The values of the strength azimuthal anchoring energy A as the function of temperatures, calculated using Eq. (21), are presented in Fig. 10. Reasonable agreement is observed between the calculated values and experimental results, which were obtained by dynamic scattering method [18].

III. CONCLUSION

In this paper, we investigated the orientational relaxation phenomena in the twisted nematic cell both for the cases of a strong and weak anchoring orientations, the relaxation times for these regimes of relaxation both in the vicinity of a nematic–smectic-A second-order phase transition temperature and far from it. The relaxation of the director $\hat{\mathbf{n}}$ to its equilibrium orientation $\hat{\mathbf{n}}_{eq}$, in the twisted nematic cell under the influence of an external electric field $\mathbf{E}$, directed perpen-

dicular to a uniformly oriented NLC, is calculated using the balance of the torques exerted on the director. The influence of the electric, elastic, and viscous torques on the orientational dynamics of the director is reflected in the relaxation of the director to its equilibrium position with the different relaxation times. With a decrease of the electric field, the magnitude of the relaxation time both for the cases of the strong and weak anchoring is increased, and the value of the relaxation time τ_R, at $E/E_{th} \sim 1.0$, in two order of magnitude higher than one, at $E/E_{th} \sim 7.0$. Notes that the value of the azimuthal angle $\phi(\tau,z)$, in the case of $E/E_{th} \sim 1.0$, relaxed to small equilibrium angle $\phi_{eq}(z)$, and the values of the later angle vary slow between 0, at the boundary of the cell, and $\sim 12°$, in the center of the cell, whereas, in the case of $E/E_{th} \sim 7.0$, the azimuthal angle relaxed to equilibrium $\phi_{eq}(z)$, which, in turn, vary rapidly between 0 and $\pi/2$. It should be pointed out that the electric field, in the case of the weak anchoring, puts the director into an equilibrium orientation, practically, with the same, as in the case of the strong anchoring, relaxation time, and the magnitude of that time decrease slow with increasing the anchoring strength. In turn, the calculations of the angular velocity $\omega(\tau,z)$ of the director in the twisted nematic cell shows, that under the action of the external electric field its magnitude is increase up to some maximum, within the first shorter relaxation term, and then, characterized by the fast decreasing of $\omega(\tau,z)$ to 0, within the second longer relaxation term. Note that the second half of the relaxation regime is often characterized by a complicate behavior of $\omega(\tau,z)$.

On the other hand, when the director rotates in the plane parallel to both glass plates, the electric, elastic, and viscous forces may excite the travelling wave spreading along the z axis, and the front of $\phi(\tau,z)$ starts to move away from the one edge of the cell to their second edge. Its asymptotic slowest velocity is proportional to the field strength E, and decay length inversely proportional to E. Taking into account that the thickness of the front propagation must be shorter than the cell length, one has the condition for the strength of the electric field which excites the front propagation. Hence, if we have $E \geqslant \pi E_{th}$, only than the wave short enough to fit in the cell length.

Calculations of the relaxation processes, in the temperature range, close to T_{NA}, e.g., at a few tens of mK from T_{NA} in the nematic phase, shows that the director distortion in the gap between two glass plates will maintain to be constant $[\phi(T_{\mathrm{NA}}) \equiv 0]$ across the sample both in the cases of the strong and weak anchoring. It is similar to the case of the strong anchoring, but in the presence only the twist elastic deformation, at temperatures far from T_{NA}. Note that the pretransitional anomalies in the behavior of the azimuthal strength of the anchoring energy A should be expected at temperatures less than $\log_{10}(T/T_{\mathrm{NA}}-1) < -4.0$. We believe that the present investigation can shed some light on the problem of the strong increase of the azimuthal anchoring energy strength in the vicinity of the nematic–smectic-A second-order phase transition temperature.

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